Solving problem 6

With Artem L.

Q: Two cyclists, A and B, have displacements 0 km and 70 km, respectively. At t = 0 they begin to cycle towards each other with velocities 15 km h^{-1} and 20 km h^{-1}, respectively. At the same time, a fly that was sitting on A starts flying towards B with a velocity of 30 km h^{-1}. As soon as the fly reaches B it immediately turns around and flies towards A, and so on until A and B meet.

a) What will the displacement of the two cyclists and the fly be when all three meet?
b) What will the distance travelled by the fly?

A: Let us first define two main keywords, and one formula that we are about to use in this problem. First of all, *displacement* is describing a movement, e.g., if a person goes 5 m to the right, then turns back, and returns to his starting point, the displacement will be 0 m, but the *distance* will be 10 m. *Distance* describes the total movement, e.g., first go forward, 5 m, and then go back, 5 m, which results 10 m.

The formula that we are about to use is based on how we define velocity, e.g. $v = \frac{\Delta x}{\Delta t}$ → $\Delta x = v\Delta t$ → $x_1 - x_0 = v \times (t_1 - t_0)$ → $x = x_0 + vt$. *(Note, I'm using x for the displacement, and d for distance).*

In order to continue solving this problem, we need to decide what velocity will be positive, and what will be negative (velocity is speed + direction). Let's say that cyclist A has a positive velocity, and B that is moving towards A, has a negative velocity. By plugging in data from the problem into the formula, we will get following:

Cyclist	The formula
A	$x_a = v_a t$
B	$x_b = 70 - v_b t$

The table at the left consist of the basic values that we got from the problem. As you might see, cyclist A has no initial displacement, but cyclist's B initial displacement is 70 m. Note that there is a minus sign because we decided to have B's velocity negative (it depends on what kind of coordinate system we are to use). We could have done it other way around, but the problem states that the zero should start from point A.

We also have a final displacement, x_{final}, which is where A and B will meet. So:

$$x_{final} = x_a$$

$$x_{final} = x_b$$

$$x_a = x_b \text{ 'where they meet'}$$

Hence, we can equate the formulas, which results (*this gives us an opportunity to get the value of t*):

$$v_a t = 70 - v_b t$$

$$v_a t + v_b t = 70$$

$$t(v_a + v_b) = 70$$

Finally,

$$t = \frac{70}{v_a + v_b}$$

We already know the velocities, so we simply insert them into the formula above.

$$t = \frac{70}{15 + 20} = \frac{70}{35} = 2\ hours$$

Now we know the time till they will meet. With other words, both need to travel 2 hours in order to reach each other. By taking cyclist A, we can calculate the displacement ($x = x_0 + vt$). I am basically plugging in the time into the formula on previous page. (*using cyclist A*)

$$x = 15 \times 2 = 30\ km$$

If we would use cyclist B formula, this is what we would get:

$$x = 70 - 20 \times 2 = 30\ km$$

Both cyclists have a displacement of 30 km, but what about the fly? Actually, it has the same displacement because we do not count know long this fly has been flying overall. We also know that displacement equals the final distance minus the initial distance. *(As in the example where we walked 5 m at right – the initial distance, and back – the final distance, which means 5-5=0 m).*

Hence,

$$x = d_{final} - d_{initial} = 30 - 0 = 30\ km$$

PART 2

In order to calculate the distance travelled by the fly, we'll simply be using this formula: $d = vt$. Just plug in what we already know, which is the t=2, v=30, therefore, $d = 30 \times 2 = 60\ m$. Done!

Good Luck!